**MATHEMATICS SPECIALIST**

**MAWA Semester 1 (Unit 3) Examination 2017**

**Calculator-assumed**

# Marking Key

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* **the end of week 8 of term 2, 2017**

**Section Two: Calculator-assumed (92 Marks)**

**Question 8(a)**

|  |  |
| --- | --- |
| Solution |  |
| Marking key/mathematical behaviours | Marks |
| * uses (or ) * evaluates correctly * writes vector eqn of line | 1  1  1 |

**Question 8(b)**

|  |  |
| --- | --- |
| Solution  and |  |
| Marking key/mathematical behaviours | Marks |
| * chose correct format for parametric equations * correct parametric equations | 1  1 |

**Question 8(c)**

|  |  |
| --- | --- |
| Solution  lies on and for some  From equation 1, . But is not a solution of equation 3.  So does not lie on |  |
| Marking key/mathematical behaviours | Marks |
| * obtains and * shows that there is no simultaneous solution for * obtains the correct answer | 1  1  1 |

**Question 9(a)**

|  |  |
| --- | --- |
| Solution    B  Q  C  R  S  D  A  P    So the opposite sides PQ and SR in the quadrilateral PQRS are parallel (and equal in length)  By a similar argument, the sides QR and PS are parallel.  Since both pairs of opposite sides are parallel, PQRS is a quadrilateral | |
| Marking key/mathematical behaviours | Marks |
| * uses * uses mid-point property to establish * shows that PQ and SR are parallel * deduces that QR and PS are parallel * states that quad is a parallelogram | 1  1  1  1  1 |

**Question 9(b)**

|  |  |
| --- | --- |
| Solution    and | |
| Marking key/mathematical behaviours | Marks |
| * draws a diagram to illustrate the situation * evaluates * determines  in terms of * shows how each of the components of the cross product are calculated * calculates each correctly | 1  1  1  1  1 |

**Question 10(a)**

|  |  |
| --- | --- |
| Solution    has roots at    So the possible roots of  are 6,    For,        So all the possible real roots of  are  or  Or using a CAS calculator    Substituting | |
| Marking key/mathematical behaviours | Marks |
| * Substitutes  into  and indicates  is a root * Applies the quadratic formula to determine the roots of   Or states them by use of a graphic calculator   * Indicates that * Determines the set of roots for * Determines the set of roots for b=1 | 1  1  1  1  1 |

**Question 10(b)**

|  |  |
| --- | --- |
| Solution  Given  For ,  Here  - so both rational  For , so  is not rational. Similarly, for .  For ,  and so  Here  - so again, both rational  Hence,  for  And for  Or using a CAS calculator | |
| Marking key/mathematical behaviours | Marks |
| * Determines the complex linear factors of  for or  in the required form * shows that are rational (by finding them) * Repeats procedure for the other value of * Shows that is irrational for  and for * States the three linear, rational factors of for and | 1  1  1  1  1 |

**Question 11(a)**

|  |  |
| --- | --- |
| Solution  Let        Alternatively,  Note that the required solution set is the perpendicular  bisector of the line segment joining the points    Plot  Draw the perpendicular bisector | |
| Marking key/mathematical behaviours | Marks |
| * Calculates the magnitude of each part and equates * Determines * Graphs the line   Alternatively,   * Plots the points * Indicates that the perpendicular bisector of the line segment joining these points is the solutions set required * Draws the bisector accurately | 1  1  1  1  1  1 |

**Question 11(b)**

|  |  |
| --- | --- |
| Solution    Draw each of the given inequalities  indicate which are dotted lines and  which include the points of the line  or circle.  Shade the regions that meet all the  conditions. | |
| Marking key/mathematical behaviours | Marks |
| * Each inequality graphed correctly (without the shading) * Correct regions that meets the conditions * Correct boundaries for these regions | 6  1  1 |

**Question 12**

|  |  |
| --- | --- |
| Solution      , and    So the 5th roots of are: | |
| Marking key/mathematical behaviours | Marks |
| * converts  in polar form correctly * Determines  and * Determines the other four values of * Represents the five values of on the Argand plane * Accurately places the roots on a circle with a scale indicating the radius | 1  1  1  1  1 |

**Question 13**

|  |  |
| --- | --- |
| Solution  Let  So | |
| Marking key/mathematical behaviours | Marks |
| * Substitutes  into given equation * Substitutes  into given equation * Compares real and imaginary parts * Determines * Determines the two values for * States values of | 1  1  1  1  1  1 |

**Question 14(a)**

|  |  |
| --- | --- |
| Solution  and lie in  so is normal to |  |
| Marking key/mathematical behaviours | Marks |
| * obtains 2 non-parallel vectors in * calculates the cross product correctly | 1  1 |

**Question 14(b)**

|  |  |
| --- | --- |
| Solution  Vector equation for is |  |
| Marking key/mathematical behaviours | Marks |
| * obtains a vector equation * obtains a Cartesian equation | 1  1 |

**Question 14(c)**

|  |  |
| --- | --- |
| Solution  At the point of intersection    i.e. i.e.  So the point of intersection has coordinates |  |
| Marking key/mathematical behaviours | Marks |
| * substitutes for in vector equation of * solves for correctly * states correct coordinates | 1  1  1 |

**Question 15(a)**

|  |  |
| --- | --- |
| Solution  and  so i.e. |  |
| Marking key/mathematical behaviours | Marks |
| * obtains the formulae for and * uses trig identity * eliminates correctly | 1  1  1 |

**Question 15(b)**

|  |  |
| --- | --- |
| Solution  The path is an ellipse. |  |
| Marking key/mathematical behaviours | Marks |
| * obtains correct answer | 1 |

**Question 15(c)**

|  |  |
| --- | --- |
| Solution  and |  |
| Marking key/mathematical behaviours | Marks |
| * correct answer for * correct answer for | 1  1 |

**Question 15(d)**

|  |  |
| --- | --- |
| Solution  and  So at (3,0), the extreme ‘easterly’ end of the path, the train is moving ‘south’, i.e. in the ‘negative’ direction of the axis. So the train is moving in a clockwise direction around the closed path. |  |
| Marking key/mathematical behaviours | Marks |
| * obtains the correct answer * gives a valid reason | 1  1 |

**Question 15(e)**

|  |  |
| --- | --- |
| Solution    So when or  and  So the velocity and acceleration are mutually perpendicular at the points |  |
| Marking key/mathematical behaviours | Marks |
| * uses * obtains correctly * obtains correct answers | 1  1  1 |

**Question 15(f)**

|  |  |
| --- | --- |
| Solution  so correct to 3 significant figures |  |
| Marking key/mathematical behaviours | Marks |
| * uses * obtains solution, correct to 3 sig. figs. | 1  1 |

**Question 15(g)**

|  |  |
| --- | --- |
| Solution  (from (c) and (f))  so  So i.e. the maximum speed is 20.5 centimetres per second |  |
| Marking key/mathematical behaviours | Marks |
| * obtains correct expression for * recognizes speed as the length of the velocity vector * obtains the correct answer | 1  1  1 |

**Question 16(a)**

|  |  |
| --- | --- |
| Solution  So the radius is 6 and the centre has coordinates |  |
| Marking key/mathematical behaviours | Marks |
| * completes the square * obtains correct radius * obtains correct coordinates of | 1  1  1 |

**Question 16(b)**

|  |  |
| --- | --- |
| Solution  Substituting in the equation of the sphere gives  i.e. i.e. i.e. (or by calculator)  so and intersect  so at the only point of intersection |  |
| Marking key/mathematical behaviours | Marks |
| * substitutes correctly * sets up an equation for t * solves for * deduces that the line and sphere intersect * solves for the coordinates and at the only point of intersection | 1  1  1  1  1 |

**Question 16(c)**

|  |  |
| --- | --- |
| Solution  If is the point of intersection ,  The vector is parallel to the line ,  And  So the direction of the line is perpendicular to the radial vector  Since is the point of intersection of and , must be tangential to at |  |
| Marking key/mathematical behaviours | Marks |
| * shows that is perpendicular to the radial vector at the point of intersection * argues that this implies the tangency property for | 1  1 |

**Question 17(a)**

|  |  |
| --- | --- |
| Solution  The midpoint of -8 and 2 on the real number line is -3. |  |
| Marking key/mathematical behaviours | Marks |
| * uses points -8,2 * obtains correct answer | 1  1 |

**Question 17(b)**

|  |  |
| --- | --- |
| Solution        Hence, |  |
| Marking key/mathematical behaviours | Marks |
| * uses endpoints of inequality * solves for * chooses correct value for | 1  1  1 |

**Question 17(c)**

|  |  |
| --- | --- |
| Solution  Graphs overlap on the interval .  The graph of  has been translated horizontally 2 units in the positive  direction, reflected in the horizontal axis and then translated vertically 4 units in the positive  direction.  Hence  and . |  |
| Marking key/mathematical behaviours | Marks |
| * recognizes that graphs overlap on given interval only * obtains -3 for p * obtains q=-2 * obtains s=4 | 1  1  1  1 |